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THE COLLAPSE OF CAVITIES PHENOMENON WITH PRESSURE FIELD CALCULATION

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Abstract. *Collapse of cavities (or bubbles) in liquids obeys a non-linear differential equation where the variables are the radius of the cavity and the velocity of the liquid/vapor interface, known as the cavity wall. Numerical simulations of the collapse of cavities are shown, using the finite difference method. Simulations were made using: water, due to its practical interest and because of the experimental data available; benzene, due to its high vapor pressure values, when compared to water; glycerin, due to its high viscosity values; mercury, due to its high surface tension and density values . One can conclude from photographic studies that a spherical symmetry hypothesis can be used for the collapse of one single cavity. This symmetry is broken only at the final stage of the collapse, where the interface velocity is very high.*

Pressure fields values are calculated in an area of 800 x 800 mm, for the case of several cavities, where the spherical symmetry no longer exists. Results are shown as pressure curves in the plane. Such calculations are new due to their general point of view, since the existing method does not take into account the physical properties of the fluid and it is available for one cavity only.

Key-words: *Cavity, Cavitation, Bubbles.*

1. INTRODUCTION

In the process of formation of bubbles in liquids, air and vapour are always trapped inside the bubbles, since the nucleation begins in a micro-bubble of air, see Hammitt (1980), and the bubble is filled by vapour as it grows. Such process begins when the pressure in the liquid reaches its vapour pressure. Therefore the presence of vapour must be taken into account as well as air. Although Poritsky (1950), studying the collapse of a bubble, considered a pressure inside the bubble, he didn't take into account the compression of vapour and air during the collapse.

When the bubble is submitted to greater values of pressure, collapse will occur. Some photographic studies of collapsing bubbles in a flow over ogives were made by Plesset (1949). The pressure field due to the collapse of bubbles in liquids can be calculated as a function of the radius of the bubble and the physical properties of the fluid, for special values of initial and boundary conditions.

Pressure field is calculated for several fluids under the hypothesis of adiabatic collapse, since there is no time for heat transfer to occur. BAZANINI et al. (1998). have shown that adiabatic hypothesis is more reliable than the isothermal hypothesis, for the simulation of the collapse of a single bubble. The choice of the fluids is based on their physical properties. Each selected fluid has a high value of one physical property, when compared to water, as follows: benzene, high value of vapour pressure; glycerin, high value of viscosity and mercury, high value of density. Therefore the influence of physical properties in the pressure field of a collapsing bubble in an incompressible liquid can be investigated.

2. BASIC EQUATIONS

2.1 Collapse equation

The basic equations of the motion of the bubble wall during the collapse of a spherical bubble in an incompressible liquid is obtained from the Navier-Stokes equation in the vectorial form below (Welty et al. (1984):

$$\rho_L \vec{B} - \nabla P = \rho_L \frac{D\vec{v}}{Dt} - \mu \nabla^2 \vec{v}\,, \tag{1}$$

where μ is the viscosity and ρ_L is the liquid density. B represents the body forces, P is the pressure and $\vec{v}$ is the velocity vector.

For a spherical bubble in an incompressible liquid, the motion is in the radial direction, and the velocity vector is:

$$\vec{v} = \vec{v}_r\,, \tag{2}$$

$$\nabla^2 \vec{v}_r = 0. \tag{3}$$

Variables involved in the process are shown in figure 1 below, where $R(t)$ is the radius of the bubble, r is a radial position in the liquid and R_0 is the initial radius of the bubble.

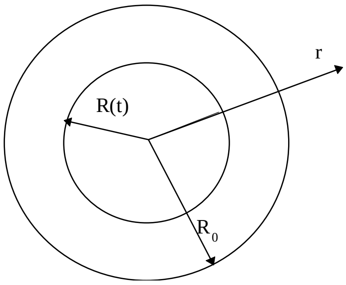

Figure 1: Variables involved in the collapse of a bubble.

Neglecting body forces, the Navier-Stokes equation yields:

$$-\frac{\nabla P}{\rho_L} d\vec{r} = \frac{D\vec{v}_r}{Dt} d\vec{r}\ . \tag{4}$$

Once the motion is in the radial direction, equation (4) can be written in the scalar form. Using the definition of substantial derivative:

$$-\frac{dP}{\rho_L} = \frac{\partial v_r}{\partial t} dr + v_r dv_r\ . \tag{5}$$

Integrating equation (5) between a radial position in the liquid r and a position far enough in the liquid (where no effects of the collapse are felt, called ∞):

$$\frac{P_\infty - P_r}{\rho_L} - \frac{v_r^{\ 2}}{2} + \int_r^\infty \frac{\partial v_r}{\partial t} dr = 0\ . \tag{6}$$

To evaluate the last term in equation (6) above, one more equation is necessary, for the radial velocity v_r as a function of time t. The continuity equation applied in a variable spherical control volume situated between the radius r and $R(t)$ (figure 1), is used, see Welty et al. (1984).

$$0 = \frac{\partial}{\partial t} \int_{VC} \rho_L\ dV + \int_{SC} \rho_L\ \vec{v}\ d\ \vec{A}. \tag{7}$$

Equation (7) above can also be obtained, as in the chapter four of Fox;McDonald (1992).

For an incompressible fluid, since ρ_L is a constant for incompressible liquids, the continuity equation becomes:

$$v_r = \frac{[R(t)]^2}{r^2} \frac{dR(t)}{dt}\ . \tag{8}$$

Evaluating the desired term, results:

$$\int_r^\infty \frac{\partial v_r}{\partial t} dr = \frac{[R(t)]^2}{r}\frac{d^2R(t)}{dt^2} + \frac{2R(t)}{r}\left[\frac{dR(t)}{dt}\right]^2. \tag{9}$$

Substituting eq. (9) in eq. (6), results:

$$\frac{v_r^{\,2}}{2} - \frac{[R(t)]^2}{r}\frac{d^2R(t)}{dt^2} - \frac{2R(t)}{r}\left[\frac{dR(t)}{dt}\right]^2 = \frac{P_\infty - P_r}{\rho_L}. \tag{10}$$

In the bubble wall:

$$r = R\,(t) \tag{11}$$

$$v_r = \frac{dR(t)}{dt} \tag{12}$$

$$P_r = P, \tag{13}$$

then equation. (10) becomes:

$$R(t)\frac{d^2R(t)}{dt^2} + \frac{3}{2}\left[\frac{dR(t)}{dt}\right]^2 = \frac{1}{\rho_L}\left[P - P_\infty\right]. \tag{14}$$

Vapor and air trapped inside the bubble are assumed to be ideal gases. Since the collapse is very fast (about 0,7 ms for water, as can be seen in Knapp & Hollander (1948)), the process is assumed as adiabatic because there is no time for heat transfer to occur.

Equations considering surface tension S and viscosity effects are used as follows:

$$P = P_e - 2\mu\frac{\partial v_r}{\partial r}, \tag{15}$$

and for the pressure external to the bubble P_e, eq. (16) below can be used:

$$P_i - P_e = \frac{2S}{R(t)}. \tag{16}$$

where the internal pressure P_i is due to the presence of vapor and air inside the bubble:

$$P_i = P_g + P_v. \tag{17}$$

Substituting equationss. (15), (16) and (17) in equation. (14), and making $R(t) = R$, $dR(t)/dt = R'$ and $d^2R(t)/dt^2 = R''$, results an equation for the collapse of a bubble:

$$RR'' + \frac{3}{2}R'^2 = \frac{1}{\rho_L}\left[P_g + P_v - P_\infty - \frac{2S}{R} - 2\mu\frac{\partial v_r}{\partial r}R\right]. \quad (18)$$

Under the hypothesis of adiabatic collapse and substituting equation (8), results:

$$RR'' + \frac{3}{2}R'^2 = \frac{1}{\rho_L}\left[\frac{P_{g_0} R_0^{3K_g}}{R^{3K_g}} + \frac{P_{v_0} R_0^{3K_v}}{R^{3K_v}} - P_\infty - \frac{2S}{R} - \frac{4(\mu_g + \mu_L)}{R}R'\right]. \quad (19)$$

Air and vapor initial pressure inside the bubble, P_{g0} and P_{v0}, respectively, shall be considered as well as air and vapor adiabatic constants, K_g and K_v.

2.2 Pressure field equations

Available method for pressure field calculation during the collapse of a bubble is appropriate for one empty bubble only, and disregards physical properties of the fluid, as can be seen in the classical work by Rayleigh (1917).

To calculate the pressure field taking into account the physical properties of the fluid, bubbles are assumed as sinks in the potential flow theory. Since eq. (19) is in the differencial form, manipulations of the Navier-Stokes and continuity equations in the differencial form are necessary to find an equation for pressure field calculation. The pressure field is calculated in an area of 800 x 800 mm. For the two-dimensional case, Navier-Stokes and continuity equations are (Swanson (1970)):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_L}\frac{dP}{dx} + \nu_L\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (20)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_L}\frac{dP}{dy} + \nu_L\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (21)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (22)$$

where u and v are the velocities in the x and y direction, respectively and ν_L is the kinematic viscosity of the liquid.

Differentiating equation (20) in regards to *y* and equation (21) in regards to *x*, subtracting the former from the later equation and simplifying using equation (22), results (cross differences disappear):

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=-2\rho_L\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}-\rho_L\left(\frac{\partial u}{\partial x}\right)^2-\rho_L\left(\frac{\partial v}{\partial y}\right)^2 \tag{23}$$

The stream function for the two-dimensional case as defined by Welty et al. (1984), is:

$$u=\frac{\partial\psi}{\partial y} \tag{24}$$

$$v=-\frac{\partial\psi}{\partial x} \tag{25}$$

Using the stream function definition above, eq. (23) becomes:

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=2\,\rho_L\left[\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}-\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2\right] \tag{26}$$

The use of the above equation can be made for any number of bubbles, as seen in Bazanini (1998). Once the bubbles are treated as sinks, the stream function field can be obtained by simple addition of the stream functions of every bubble.

Stream function field calculations for every sink can be made using the following equation,:

$$\psi=-R\,R^{'}\,C \tag{27}$$

where *C* is the position of the calculated point related to the sink.

3. RESULTS

To calculate the pressure field it is necessary to solve equations (19) and (26). An analytical solution is very difficult or may be impossible. Once numerical methods become necessary, equations (19) and (26) are solved using the finite difference method, for the following conditions: R_0 = 3,56 mm, P_{g0} = 40 Pa (initial conditions) as measured by Knapp & Hollander (1948); P_∞ = 50,000 Pa as boundary condition. The time step has been made equal to 10^{-5} s, which is shown to be enough for such calculations. Calculations are ended when the bubble radius reaches 1 mm, since it is not possible to assure the existence of the bubble beyond this value.

Equations (19), (26) and (27) take into account the physical properties of the fluid and can be used for several bubbles.

In table 1 below, values of initial vapour and air pressure are shown. Vapour pressure is expected to have a great influence in the pressure field, as the vapour and air are compressed when the bubble collapses, vapour and air pressures inside the bubble raise, and initial vapour pressure is much greater than initial air pressure, as can be seen in table 1.

Table 1. Initial air and vapour pressure

Fluid (vapour)	P_0 (Pa)
Benzene	10,000
Glycerin	0.014
Mercury	0.17
Water	2,340
Air	40

Since the calculation method presented here is adequate in the presence of several bubbles, it is used for four randomly disposed bubbles in an area of 800 x 800 mm, under the initial and boundary conditions:P_{g0} = 40 Pa; R_0 = 3,56 mm and P_∞ = 50,000 Pa. For four bubbles there is no spherical symmetry in the pressure field anymore, so the pressure field equation (equation (26)) will be in the rectangular form._Results are shown in figures 2 to 5 for each fluid:

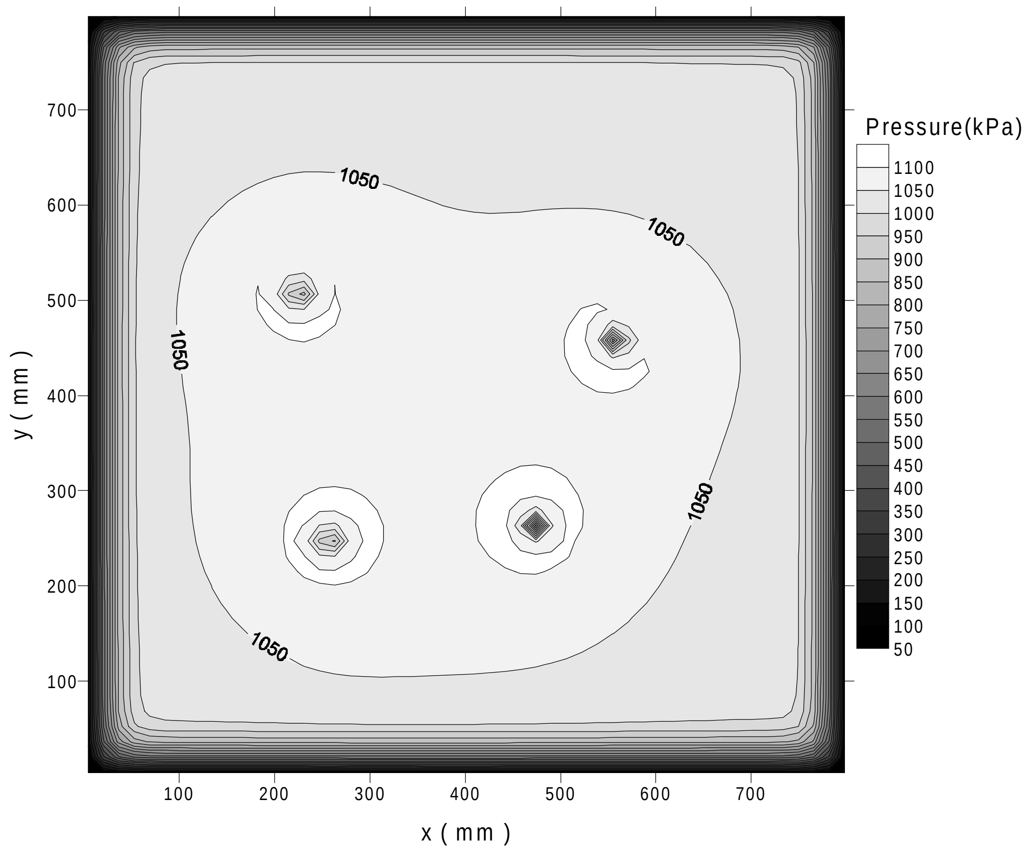

Figure 2. Pressure field for water.

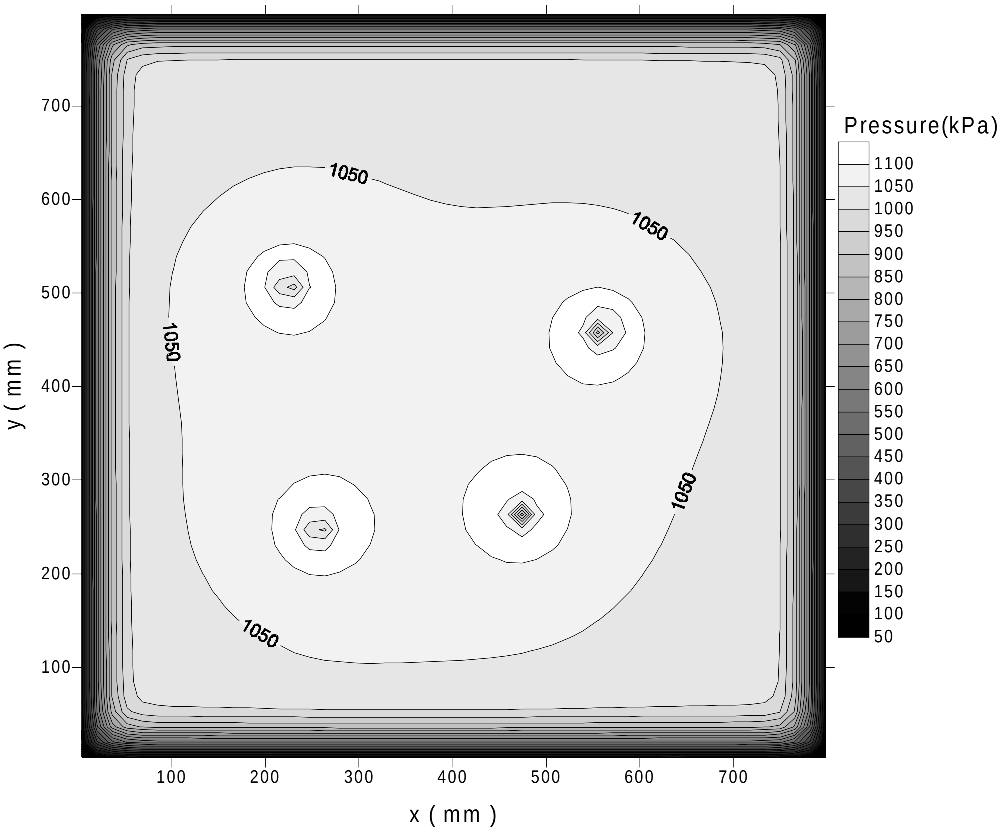

Figure 3. Pressure field for benzene.

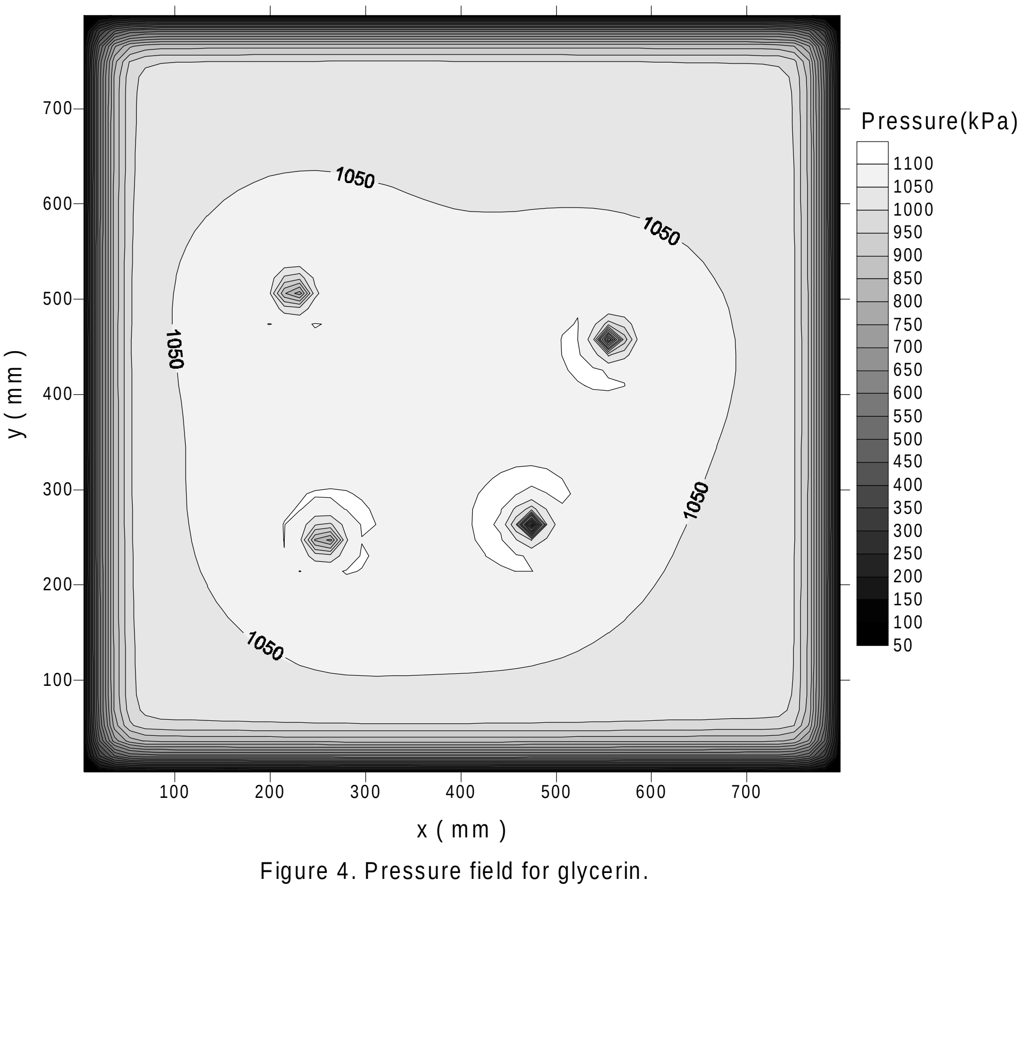

Figure 4. Pressure field for glycerin.

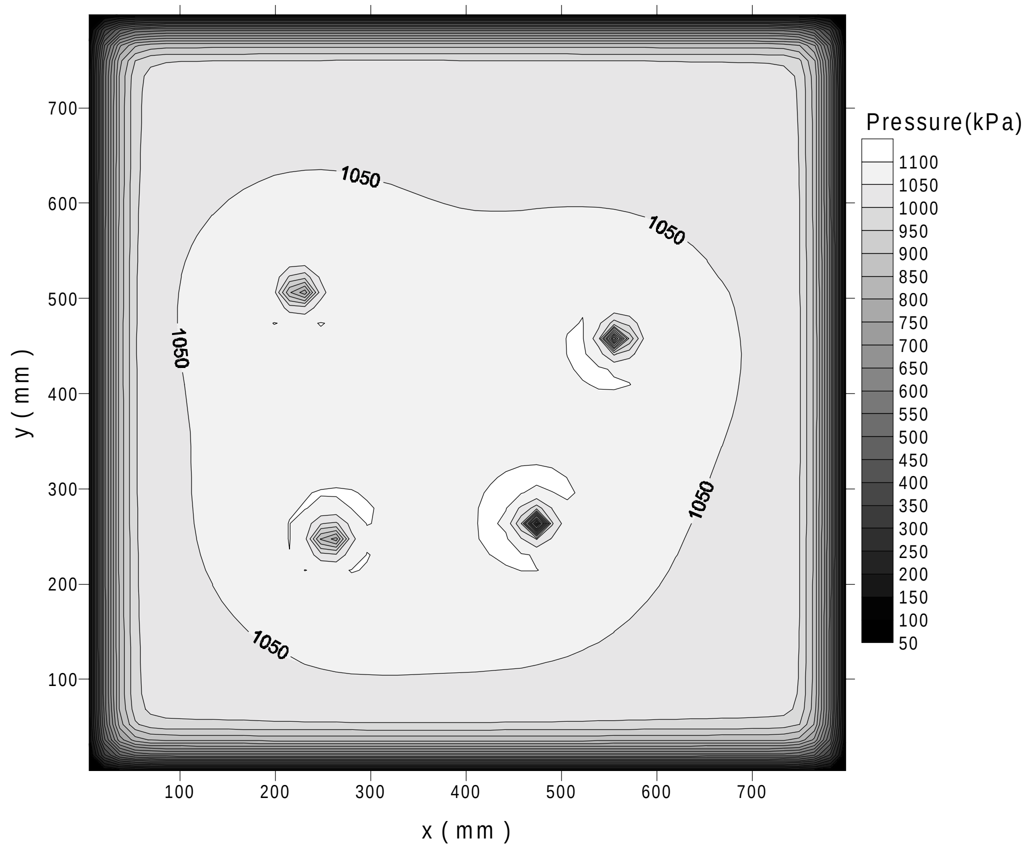

Figure 5. Pressure field for mercury.

It can be seen in figures 2 to 5 that pressure values are greater for benzene and water respectively, because these fluids have the greatest values of initial vapour pressure. Although the scale is the same in figs. 2 to 5 (1100 kPa), one can see that benzene has greater values of pressure in the field (white areas), followed by water, due to initial vapour pressure values, which raise as the collapse proceeds. Greater values of pressure are observed in the liquid (white areas), because there we have the effect of the collapse of all bubbles at the same time. Inside the bubbles, pressure values are smaller for glycerin and mercury (see figures 2 to 5) than for water and benzene, due to initial vapour pressure values, since viscosity and density didn't take a major influence in the pressure field.

4. CONCLUSIONS

Initial vapour pressure was of major influence in the pressure field of collapsing spherical bubbles in an incompressible liquid as can be seen in figures 2 to 5. This fact is due to the compressing process during the collapse of the bubble. As the bubble collapses, vapour and air are compressed and pressure inside the bubble raises. Since water was used as a parameter, one can conclude from figs. 4 and 5 that properties like viscosity and density are not of major influence for the pressure field.

Greater values of pressure were find in the liquid, because of the simultaneous influence of all collapsing bubbles. Glycerin and mercury have smaller values of pressure than water and benzene, due to initial vapour pressure values.

Calculations presented here are adequate to any fluid of known physical properties, but there are is infinite number of possibilities for calculations when varying parameters such as number and positions of bubbles, initial and boundary conditions, and physical properties.

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